Math 307: Diff. Eq. - Dr. Loveless

<u>Essential Course Info</u>

My Course Website: math.washington.edu/~aloveles/

Homework Log-In (use UWNetID): webassign.net/washington/login.html

<u>First week to do list</u>

1. Read 1.1-1.3, 2.1-2.2 of the book. Start attempting HW.
2. Be ready for an entry task on Friday
   one integral (like handout), and one short diff. eq. set-up (like handout)

Today

- Syllabus/Intro
- Section 1.1-1.3
  - diff. eq. examples
  - checking a solution

<u>Week 1 assignments</u>

Closing time is always 11:59pm.

- HW 1 closes Mon (1.1-1.2)
- HW 2 closes next Fri (1.3, 2.2)

What we will do in this course:
Learn terminology, examples, and solving methods for 1st and 2nd order ordinary differential eqns.

1. **Ch. 1 – Intro/Applications**
   - Applications
   - Slope/direction fields
2. **Ch. 2 – First Order ODE's**
   - Separable equations
   - Linear eqs (integrating factor)
   - Equilibrium solns
   - Euler's method
3. **Ch. 3 – Second Order ODE's**
   - Characteristic equations
   - Undetermined coefficients
   - Applications to oscillators
4. **Ch. 6 – LaPlace Transforms**
   - Important systematic method for solving differential equations

Chapter 1: Intro to Diff. Equations

Some terminology

- A *differential equation* is any equation involving a derivative.

- An *ordinary differential equation (ODE)* is an equation involving derivatives relating only two variables (dependent/independent).

- A *partial differential equation (PDE)* is an equation involving partial derivatives.

We only study ODE's in this course.

- The *order* of an ODE is the highest derivative that appears in the equation.

Checking Solutions *(like 4-6 in HW)*

An explicit *solution* to a differential equation is a function that satisfies the equation.

Example 1: $y' - 4y = 0$ has one solution that look like $y(t) = e^{rt}$. Find *r*.

Example 2: $y'' - \frac{6}{t^2}y = 0$ has two solutions that look like $y(t) = t^r$. Find the two values of *r*.

Example 3: $y'' - y' = 3x$ has one solution that look like

$y(t) = e^x + rx + sx^2$. Find *r* and s.

Applications/Units (7-10 in HW)

$$\frac{dy}{dt} =$$

"rate of change of y with respect to t"

Example:

Let $P(t)$ =“population after t years”.

Assume the population grows at a rate proportional to its size. That is,

$$\frac{dP}{dt} = kP$$

Units Discussion:

What if P is in thousands of people?

Such as $P_2(t) = \frac{P(t)}{1000}$ thousand people. What is $\frac{dP_2}{dt}$?

What if t is in days instead of years?

Such at $t_2 = 365t$ days.

What is $\frac{dP}{dt_2}$?

Some Motivation

Air Resistance Example:

Assume an object with mass m is dropped from 1000 meters with an initial velocity of $v(0) = 0$ m/s.

Recall: Newton's Law

$$ma = F \quad (Force)$$

If we write $a(t) = v'(t) = h''(t)$, then we see this leads to an ODE.

1. No air resistance, then the only force is gravity F = -mg (g = 9.8 m/s^2).

$$mv' = F_g = -mg$$

***I**nitial **V**alue **P**roblem* (IVP)

$$v' = -g$$
$$v(0) = 0$$

2. Assume air resistance exerts a force proportional to speed in the opposite direction.

$$mv' = F_g + F_A = -mg - rv$$

So

$$v' = -g - \frac{r}{m}v$$

$$v(0) = 0$$

Mass-Spring Example:

Assume an object with mass m is attached to a spring that is attached to a wall. Natural length is the distance from the wall at which the mass is at rest (no stretch or push force).

Let x = “the distance the spring is stretched beyond natural length”.

Hooke’s Law says force due to the spring is proportional to x and in the opposite direction. In other words,

$$Force = -kx$$

So, once again, if you want to model the motion of the mass after you stretch it and let go, you use Newton's Law:

$$ma = F = -kx$$

and $a(t) = x''(t)$

So

$$m\, x'' = -kx$$

It turns out that one solution to this is $x(t) = cos(\omega t)$

Aside: This is called a "simple harmonic oscillator" and ω is the "natural frequency" (radians/time). And $2\pi/\omega$ is the wavelength (time from peak-to-peak)

Circuits Example: Kirchoff's laws observe that the sum of the voltage drops in a circuit equals zero (source, resistance, capacitance, inductance).

So $-V_0 + V_R + V_C + V_L = 0$

with

$$I = \frac{dq}{dt} = q'$$

$$V_R = RI = Rq'$$

$$V_C = \frac{q(t)}{C} = \frac{q}{C}$$

$$V_L = L\frac{dI}{dt} = Lq''$$

So $V_0 = V_R + V_C + V_L$

$$V_0 = Rq' + \frac{q}{C} + Lq''$$